Student Name: ______________________________

Student Number: ____________________________

GE 213.3 - Mechanics of Materials

MIDTERM EXAMINATION

March 2, 2005

Quest. 1: _____	
Quest. 2: _____	
Quest. 3: _____	
Quest. 4: _____	

Professor: B. Sparling

Time Allowed: 3 Hours

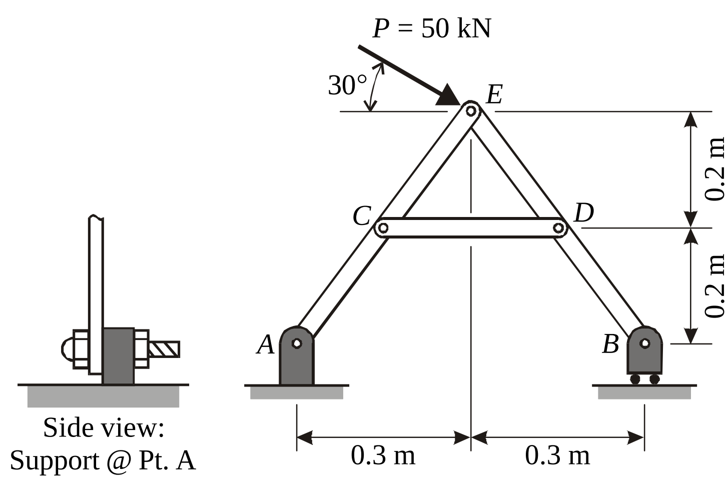
Notes:
- Closed book examination; Calculators may be used
- The value of each question is provided along the left margin
- Supplemental material is provided at the end of the exam (formulas)
- Show **all** your work, including all formulas, calculations and **units**
- Write your work in the space provided on the examination sheet. (The backs of the examination sheets may also be used if required)

MARKS

25 **<u>QUESTION 1:</u>** The three member assembly shown below has a pinned support at Point *A* and a roller support at Point *B*. An inclined force, $P = 50$ kN, is applied at Point *E*. All connections in the assembly are pinned connections.

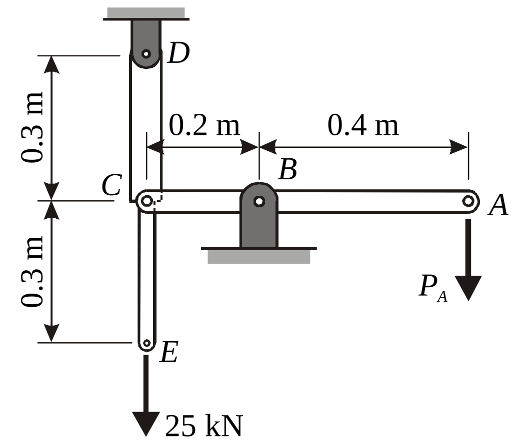
a) If the pin at Point *A* has a diameter of 25 mm and is fabricated from a material with an ultimate shear strength of $\tau_U = 180\ \text{MPa}$, determine the factor of safety for that pin.

b) If the central portion of member *CD* (i.e. away from the pins at either end) has a square cross-section and is fabricated from a material with an allowable normal stress of $\sigma_{all} = 150\ \text{MPa}$, determine the required cross-sectional dimensions for the central portion of member *CD*.



25 **QUESTION 2:** Rigid bar *ABC* is supported at Point *B* and connected to the vertical stepped bar *DCE* at Point *C*. All connections are pinned connections. Stepped bar *DCE* is continuous from Point *D* to Point *E* and is fabricated from material with an elastic modulus of E = 150,000 MPa and has a cross-sectional area of 500 mm^2 from Points *D* to *C*, and a cross-sectional area of 200 mm^2 from Points *C* to *E*.

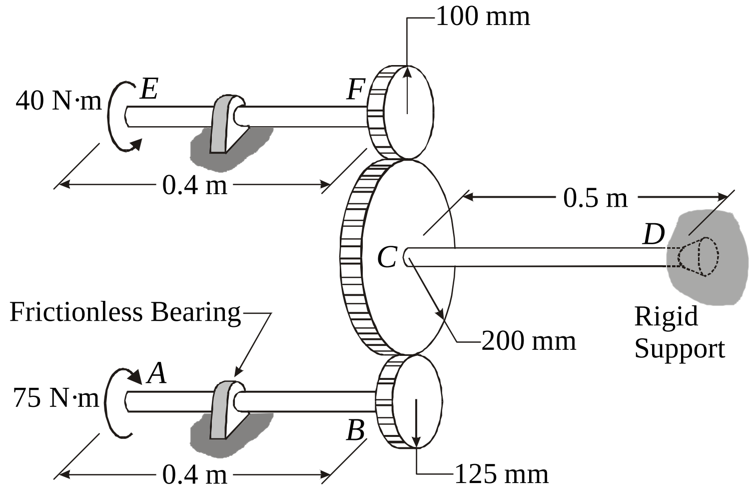
If a vertical force of 25 kN is applied at Point E as shown, determine the magnitude of the vertical force P_A applied at Point *A* so that the vertical displacement of Point *E* will be zero.



25 **QUESTION 3:** Three solid shafts (*AB*, *CD* and *EF*), each of which has a diameter of 30 mm, are connected by three gears of different sizes, as shown below. End *D* of shaft *CD* is rigidly attached to a support that prevents rotation. External torques are applied at the free ends *A* and *E*. The shafts are fabricated from a material with a modulus of rigidity $G = 77{,}000$ MPa .

a) Determine the maximum shear stress in shaft *CD*.

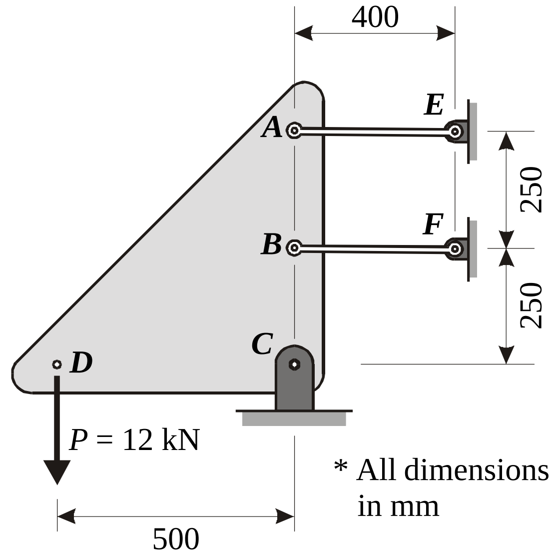
b) Calculate the magnitude and direction of the angle of twist at Point *E*.



25 **QUESTION 4:** Rigid triangular plate *ABCD* has a pinned support at Point *C* and is connected to two horizontal rods *AE* and *BF* that have pinned connections at both ends. The rods have a cross-sectional area of 50 mm^2, an elastic modulus of $E = 125$ GPa, and a coefficient of thermal expansion of $a = 23\times10^{-6}/°C$; they are unstressed prior to the application of the vertical load $P = 12$ kN at Point *D*. If, after the application of load *P*, the temperature of both rods is decreased by 20° C, determine:

a) The axial force in Rod *AE*; and

b) The horizontal displacement at Point *A*.



Supplemental Material:

- **Static Equilibrium:** $\Sigma F_x = 0 ; \Sigma F_y = 0 ; \Sigma F_z = 0$ & $\Sigma M_x = 0 ; \Sigma M_y = 0 ; \Sigma M_z = 0$
- **Normal Stress:** $s_{avg} = \dfrac{P}{A}$ $F = \int_A s \, \mathrm{d}A$
- **Direct Shear Stress:** $t_{avg} = \dfrac{V}{A}$ (Single) or $t_{avg} = \dfrac{V}{2A}$ (Double)
- **Bearing Stress:** $s_b = \dfrac{P}{t\, d}$
- **Allowable Stress:** $F.S. = \dfrac{P_U}{P_D}$ or $F.S. = \dfrac{s_U}{s_D}$ **;** $s_{all} = \dfrac{s_U}{F.S.}$ $P_{all} = s_{all} \, A$ $A_{req} = \dfrac{P_D}{s_{all}}$
- **Stresses on Oblique Planes:** $s_q = \dfrac{P \cos q}{A_o / \cos q} = \dfrac{P}{A_o} \cos^2 q$ **;** $t_q = \dfrac{P \sin q}{A_o / \cos q} = \dfrac{P}{A_o} \sin q \cos q$
- **Average Normal Strain:** $e = \dfrac{d}{L_o} = \dfrac{L^* - L}{L}$
- **Hooke's Law:** $s = E\, e$
- **Axial Deformations:** $d = \dfrac{P\, L_o}{A_o\, E}$**;** $d_{tot} = \sum_i \dfrac{P_i\, L_i}{A_i\, E_i}$**;** $d = \int_0^L \dfrac{P(x)}{A(x)\, E(x)} \, \mathrm{d}x$
- **Thermal Deformations:** $d_T = a\, (\Delta T)\, L_o$ $e_T = \dfrac{d_T}{L_o}$
- **Poisson's Ratio:** $e_y = e_z = -n\, e_x$ $e_y = e_z = -\dfrac{n\, s_x}{E}$
- **General Hooke's Law:** $e_x = \dfrac{s_x}{E} - n \dfrac{s_y}{E} - n \dfrac{s_z}{E}$ **;** $e_y = -n \dfrac{s_x}{E} + \dfrac{s_y}{E} - n \dfrac{s_z}{E}$ **;** $e_z = -n \dfrac{s_x}{E} - n \dfrac{s_y}{E} + \dfrac{s_z}{E}$
- **Shearing Strain & Stress:** $q^* = \dfrac{p}{2} - g_{xy}$**;** $g_{xy} = \dfrac{t_{xy}}{G}$; $g_{yz} = \dfrac{t_{yz}}{G}$; $g_{zx} = \dfrac{t_{zx}}{G}$**;** $G = \dfrac{E}{2(1+n)}$
- **Resultant Torque:** $T = \int_A r\, t \, \mathrm{d}A$
- **Torsional Strains:** $g = \dfrac{r\, f}{L}$ $g_{\max} = \dfrac{c\, f}{L}$ $g = \left(\dfrac{r}{c} \right) g_{\max}$
- **Torsional Stresses:** $t = \left(\dfrac{r}{c} \right) t_{\max}$ $t_{\max} = \dfrac{T\, c}{J}$ $t = \dfrac{T\, r}{J}$ $J = \int_A r^2 \, \mathrm{d}A = \dfrac{p}{2} c^4$
- **Torsional Angle of Twist:** $f = \dfrac{T\, L}{J\, G}$
- **Torsion - Gear Compatibility:** $\phi_1 \, \rho_1 = \phi_1 \, \rho_2$
- **Pure Bending - Normal Strain:** $e_x = -\dfrac{y}{r}$ $e_{\max} = c/r$ $e_x = -\dfrac{y}{c} e_m$
- **Pure Bending - Normal Stress:** $s_x = -\dfrac{y}{c} s_m$ $s_x(y) = -\dfrac{M\, y}{I}$ $s_{\max} = \dfrac{M\, c}{I}$
- **Bending – Section Properties:** $I = \int_A y^2 \, \mathrm{d}A$**;** Centroid: $\int_A y \, \mathrm{d}A = 0$
- **Properties of Composite Areas:** $\bar{y}\, A = \sum_i y_i\, A_i$ $I = \sum_i \left(I_i + A_i\, d_i^2 \right)$